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NATURAL THERMAL-CONCENTRATION CONVECTION

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Combined thermal and concentration convection is studied for the example of the problem of flow and transport near the surface of a horizontal disk.

Natural convection processes in the field of gravity are determined by the dependence of the density of the fluid on the temperature or the concentration of the impurity diffusing in it and the nonuniformity of the fields of these quantities in the presence of heat and mass transfer from surfaces immersed in the fluid. There are a very large number of studies of free-convection stimulated by only one of the indicated factors and of the corresponding convective transport processes (see the review in [1, 2]). A significant number of numerical studies of situations in which both factors are important at the same time have also been performed. Free convective flow at a horizontal surface is, however, an exception in this respect; to analyze this flow it is necessary to study not only the horizontal but also the vertical component of the vector equation of conservation of momentum. This makes the calculations significantly more complicated, which apparently explains the fact that there are only a few isolated papers on the study of such flow.

Numerical solutions, however, in spite of their importance in obtaining reliable quantitative results, are very cumbersome and, most importantly, they are not very useful for constructing a complete physical picture of the process and formulating comparatively simple relations describing the process in a wide range of values of the parameters. Attempts, of which we are aware, to describe analytically the combined thermal concentration convection (made, in particular, in the analysis of the macrokinetics of heterogeneous reactions) are, as a rule, based on the use of asymptotic boundary layer methods combined with the principle of superposition, which cannot, in principle, be correct when it is applied to strong convective heat conduction and diffusion processes [3]. Inaccuracies of a fundamental character, concerning the determination of the effective thicknesses of the hydrodynamic and thermal or diffusion layers (see below), which must be corrected, are also encountered in the use of a thin boundary layer. Finally, there are experimental indications [4] of the

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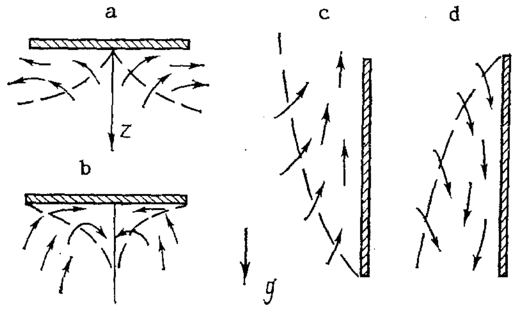

Fig. 1. Schematic diagram of two types of convective flow at the bottom surface of a disk (a, b) and at a vertical plate (c, d); the dashed lines show provisionally the boundary of the hydrodynamic boundary layer.

possibility of a change in the regime of thermal-concentration convection (change in the direction of flow), which still has not been given a theoretical-physical explanation.

Natural convection processes, in which both mechanisms of the change in density operate, are very common in cases when evaporation, condensation, or dissolution occurs on a surface or some heterogeneous reaction with a significant heat effect occurs. The latter situation is especially important in application to organization of diverse electrochemical processes, when the thermal factors can affect the magnitude of the flows of reagents (and therefore also the macrokinetics of the process) and change the direction of the convective motion itself. This situation unfortunately is usually ignored not only in chemical or electrochemical technology but also in the formulation and interpretation of experiments, and this leads to incorrect conclusions (see, for example, [4]).

In what follows the problem of combined thermal-concentration convection is studied for the example of flow at a horizontal disk, participating simultaneously in both heat and mass exchange with the surrounding liquid. For the case when the bottom surface of the disk is active the types of flows shown schematically in Fig. 1 are possible; if the top surface is active, the flow patterns are completely analogous. Systems of this type were employed in [5-10] for studying purely thermal free convection. The visualization of the flow, made in a number of works (see, for example, [8]), confirms the schemes presented and indicates that a thin thermal boundary layer forms on the surface of the disk. This serves as an adequate experimental foundation for applying the existing methods of the boundary-layer approximation, in particular, in this work also.

We shall first examine the situation when as a result of the combined effect of thermal and concentration factors the density of the fluid is lower than far away from the disk (see Fig. 1a). In this case the point r = 0, z = 0 is the point of incidence of the flow, from which a hydrodynamic boundary layer, whose thickness continually increases, extends in the radial direction. The equations of hydrodynamics and convective heat and mass transfer in the Boussinesq approximation have in this case the following form:

$$\begin{gathered}\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial v}{\partial z}=0,\quad u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}=-\frac{1}{\rho_0}\frac{\partial p}{\partial r}+\nu\frac{\partial^2 u}{\partial z^2},\\ 0=-\frac{1}{\rho_0}\frac{\partial p}{\partial z}+g\,[\beta_T(T-T_0)+\beta_c(c-c_0)],\\ u\frac{\partial T}{\partial r}+v\frac{\partial T}{\partial z}=a\frac{\partial^2 T}{\partial z^2},\quad u\frac{\partial c}{\partial r}+v\frac{\partial c}{\partial z}=D\frac{\partial^2 c}{\partial z^2},\\ \beta_T=\frac{1}{\rho_0}\left(\frac{\partial\rho}{\partial T}\right)_0,\quad \beta_c=\frac{1}{\rho_0}\left(\frac{\partial\rho}{\partial c}\right)_0,\end{gathered}\tag{1}$$

and the derivatives in the definitions of β_T and β_c are calculated with $\rho = \rho_0$. We neglect the dependence of the kinematic viscosity, the thermal diffusivity, and the diffusion coefficient on the temperature and concentration, though, as shown in [11], in many cases this can lead to significant uncertainties in the determination of the heat and mass fluxes. The solution of Eqs. (1) must satisfy the following boundary conditions:

$$
\begin{gathered}
z=0,\ r\leqslant R:\ u=v=0,\ T=T_w,\ c=c_w,\\
z=0,\ r\geqslant R:\ \partial T/\partial z=0,\ \partial c/\partial z=0,\\
z\to\infty:\ p\to 0,\ u\to 0,\ T\to T_0,\ c\to c_0
\end{gathered}
\tag{2}
$$

(p is the pressure perturbation, determined by the convective motion). The solution of the problem (1) and (2) is very complicated and at the present time can be constructed only numerically. Here, using the ideas of boundary-layer theory, we obtain an approximate self-similar solution, for which the radius of the disk R must be made to pass formally to infinity. Introducing the parameters

$$
\mathrm{Gr}_T=\frac{g|\beta_T|R^3|T_w-T_0|}{\nu^2},\quad \mathrm{Gr}_c=\frac{g|\beta_c|R^3|c_w-c_0|}{\nu^2}
\tag{3}
$$

and the self-similar variables

$$
\begin{gathered}
\eta=\frac{z}{r}\left[\frac{\mathrm{Gr}_T}{5}\left(\frac{r}{R}\right)^3\right]^{1/5},\ \theta(\eta)=\frac{T-T_0}{T_w-T_0},\ \varphi(\eta)=\frac{c-c_0}{c_w-c_0},\\
u=\frac{1}{r}\frac{\partial\psi}{\partial z},\ v=-\frac{1}{r}\frac{\partial\psi}{\partial r},\\
\psi=5\nu r\left[\frac{\mathrm{Gr}_T}{5}\left(\frac{r}{R}\right)^3\right]^{1/5}f(\eta),\ p=\frac{5\nu^2\rho_0}{r^2}\left[\frac{\mathrm{Gr}_T}{5}\left(\frac{r}{R}\right)^3\right]^{4/5}G(\eta),
\end{gathered}
\tag{4}
$$

we obtain from (1) and (2) the following problem:

$$
\begin{gathered}
5f'''+8ff''-f'^2=2(G-\eta G'),\ G'=-\theta-\varkappa\varphi,\\
\theta''+(8/5)\,\mathrm{Pr}\,f\theta'=0,\ \varphi''+(8/5)\,\mathrm{Sc}\,f\varphi'=0,\\
f(0)=f'(0)=0,\ \theta(0)=\varphi(0)=1,\\
G(\infty)=0,\ f'(\infty)=0,\ \theta(\infty)=\varphi(\infty)=0,
\end{gathered}
\tag{5}
$$

where the following dimensionless parameters have been introduced:

$$
\mathrm{Pr}=\frac{\nu}{a},\ \mathrm{Sc}=\frac{\nu}{D},\ \varkappa=\frac{\beta_c(c_w-c_0)}{\beta_T(T_w-T_0)}.
\tag{6}
$$

The situation shown in Fig. 1a in the case of purely thermal convection and $\beta_T < 0$ obviously corresponds to $T_w > T_0$. If $\kappa > 0$, then a nonuniformity of the concentration gives rise to a buoyancy force acting in the same direction as the buoyancy force owing to the nonuniformity of the temperature, i.e., upwards. When $\kappa < 0$, the first force weakens the second force. A problem similar in form to (5) was formulated previously in [12].

The expansion of the function $f(\eta)$ for small values of η has the form $(\alpha/2)^2\eta + O(\eta^3)$. According to the considerations described in detail in [13], in solving the third and fourth equations in(5) in the case Pr ≫ 1 and Sc ≫ 1 it is admissible to replace f by the leading term in this expansion. Then we have

$$
\theta=1-\left[\frac{1}{3}\Gamma\left(\frac{1}{3}\right)\right]^{-1}\int_0^{k\eta}\exp(-t^3)\,dt,\ k=\left(\frac{4\alpha\,\mathrm{Pr}}{15}\right)^{1/3},
$$

and a completely analogous formula with Pr replaced by Sc is obtained for $\phi(\eta)$. A further simplification can be made by using the first term of the expansion for the integrand, which gives

$$
\theta=1-\frac{\eta}{\eta_T},\ \eta<\eta_T;\ \theta=0,\ \eta>\eta_T;\ \eta_T=0{,}89\left(\frac{15}{4\alpha\,\mathrm{Pr}}\right)^{1/3},
$$

and in addition η_T is the thickness of the thermal boundary layer. It is obvious that this procedure somewhat reduces the buoyancy forces stimulated by the temperature nonuniformity. For this reason it is more logical to determine η_T from some integral relation (as is generally characteristic of the boundary-layer theory), for which it is natural to study the equation for the zeroth moments of the starting and approximate functions $\theta(\eta)$, i.e.,

$$\int_0^\infty \theta(\eta)\, d\eta = \frac{1}{2}\eta_T,$$

where $\theta(\eta)$ is determined above in terms of a quadrature. A simple calculation using a computer gives the following formulas:

$$\theta = \begin{cases} 1 - \eta/\eta_T, & \eta \leqslant \eta_T \\ 0, & \eta \geqslant \eta_T \end{cases}, \quad \varphi = \begin{cases} 1 - \eta/\eta_c, & \eta \leqslant \eta_c \\ 0, & \eta \geqslant \eta_c \end{cases}, \tag{7}$$

$$\eta_T = 1.01\,(15/4\alpha\,\mathrm{Pr})^{1/3}, \quad \eta_c = 1.01\,(15/4\alpha\,\mathrm{Sc})^{1/3}.$$

For liquids Sc ≫ 1 always holds, but Pr can vary over a wide range. The requirement Pr ≫ 1 obviously limits the class of solutions that can be studied, but it may be assumed with confidence that Sc > Pr and therefore $\eta_c < \eta_T$. We emphasize that a serious error is made in [13] as well as in very many subsequent works: it is assumed without foundation that the thickness of the hydrodynamic boundary layer is equal to η_T or η_c, depending on whether thermal or concentration convection is under study. In reality, even a concentrated force, formally corresponding to a thermal or diffusion boundary layer with vanishingly small thickness, generates motion in a finite region, and in addition the outer boundary of this region approaches infinity as the modulus of the force approaches zero. According to this assumption, the boundary conditions in the limit $\eta \to \infty$ in (5) are replaced by conditions at $\eta = \eta_T$ or $\eta = \eta_c$. In the present analysis of combined convection the inadequacy of this assumption already becomes obvious because η_T and η_c are completely equivalent and it is not clear which of these values of η should be employed as the boundary value.

Thus the determination of the heat and mass fluxes is actually reduced to calculating the coefficient a, for which an equation with boundary conditions for f in (5) with $G(\infty) = 0$ and

$$G' = \begin{cases} -(1 - \eta/\eta_T) - \varkappa(1 - \eta/\eta_c), & 0 \leqslant \eta \leqslant \eta_c, \\ -(1 - \eta/\eta_T), & \eta_c \leqslant \eta \leqslant \eta_T, \\ 0, & \eta \geqslant \eta_T, \end{cases} \tag{8}$$

must be solved.

For the solution we shall employ the method of joined asymptotic expansions [14]. Taking into account the fact that for $\eta \ll 1$ the conditions $f' \sim \eta \ll 1$, $f \sim \eta^2 \ll 1$ hold we obtain for the region indicated

$$\begin{gathered} f = \frac{\alpha}{2}\eta^2 - \frac{1}{150}\left(\frac{1}{\eta_T} + \frac{\varkappa}{\eta_c}\right)\eta^5, \quad 0 \leqslant \eta \leqslant \eta_c, \\ f = A^* + B^*\eta + \frac{\alpha^*}{2}\eta^2 - \frac{1}{150}\frac{\eta^5}{\eta_T}, \quad \eta_c \leqslant \eta \leqslant \eta_T, \\ f = A + B\eta + C\eta^2, \quad \eta_T \leqslant \eta \ll 1, \end{gathered} \tag{9}$$

where new constant coefficients which can be expressed in terms of α and η_T, η_c from the six conditions of continuity of f and its derivatives on the planes $\eta = \eta_T$ and $\eta = \eta_0$ have been introduced.

In the limit $\eta \to \infty$ the function $f(\eta)$ approaches a constant m, characterizing the total flux of liquid along the horizontal surface. For this reason it is easy to see that for sufficiently large values of η the first two terms on the left side of the equation for f in (5) are equally significant, but the third term can be neglected. Then we have

$$f = m + n\exp[-3m(\eta - \eta_T)], \quad \eta \gg \eta_T, \tag{10}$$

and in addition the parameters m and n are unknown and must be determined, like the quantity α, from the joining conditions. The last relation in (9) plays the role of an inner asymptotic expansion while (10) is an outer expansion, written in inner variables [14]. Expanding (10) in a series in powers of $\eta - \eta_T$ and equating its coefficients to the quantities A, B, and C from (9), after calculations we obtain the equations

$$m+n=\frac{1}{2}\left(\alpha-\frac{2}{15}\varkappa\eta_c^2\right)\eta_T^2-\frac{1}{150}\eta_T^4+\frac{19}{150}\varkappa\eta_c^4,$$

$$-3mn=\left(\alpha-\frac{1}{2}\varkappa\eta_c\right)\eta_T-\frac{1}{30}\eta_T^3+\frac{1}{10}\varkappa\eta_c^3,$$

$$\frac{9}{2}m^2n=\alpha-\frac{2}{15}(\eta_T^2+\varkappa\eta_c^2),$$

from which it follows, in particular, that $m \sim n \sim \eta_{T,c}^{3/2}$. For this reason from the last equation neglecting terms of order $\eta_{T,c}^{9/2}$ we obtain

$$\alpha=\frac{2}{15}(\eta_T^2+\varkappa\eta_c^2)=0{,}507\left(\frac{1}{\mathrm{Pr}^{2/3}}+\frac{\varkappa}{\mathrm{Sc}^{2/3}}\right)^{3/5}, \qquad (11)$$

which determines the thicknesses of the thermal and diffusion boundary layers introduced in (7).

The local heat flux from the surface is

$$j_T=-\lambda\left.\frac{\partial T}{\partial z}\right|_{z=0}=\frac{0{,}37\lambda(T_w-T_0)}{R^{3/5}r^{2/5}}\mathrm{Pr}^{1/3}\left(\frac{1}{\mathrm{Pr}^{2/3}}+\frac{\varkappa}{\mathrm{Sc}^{2/3}}\right)^{1/5}\mathrm{Gr}_T^{1/5}. \qquad (12)$$

Integrating (12) over the surface of a disk with finite radius we have

$$\mathrm{Nu}=0{,}463\,\mathrm{Pr}^{1/3}\left(\frac{1}{\mathrm{Pr}^{2/3}}+\frac{\varkappa}{\mathrm{Sc}^{2/3}}\right)^{1/5}\mathrm{Gr}_T^{1/5}. \qquad (13)$$

In the absence of a concentration factor ($\kappa = 0$) we obtain from here the well-known formula $\mathrm{Nu} \sim (\mathrm{Pr}\mathrm{Gr}_T)^{1/5}$, found by the method of integral relations in [5] and later confirmed in [15, 16].

Analogously the local diffusion flux is

$$j_c=-D\left.\frac{\partial c}{\partial z}\right|_{z=0}=\frac{0{,}37D(c_w-c_0)}{R^{3/5}r^{2/5}}\mathrm{Sc}^{1/3}\left(\frac{1}{\mathrm{Pr}^{2/3}}+\frac{\varkappa}{\mathrm{Sc}^{2/3}}\right)^{1/5}\mathrm{Gr}_T^{1/5}. \qquad (14)$$

Transforming this formula with the help of (3) and (6) and integrating over the surface of the disk we obtain

$$\mathrm{Sh}=0{,}463\,\mathrm{Sc}^{1/3}\left(\frac{1}{\mathrm{Sc}^{2/3}}+\frac{1}{\varkappa\,\mathrm{Pr}^{2/3}}\right)^{1/5}\mathrm{Gr}_c^{1/5}, \qquad (15)$$

which in the absence of the thermal factor reduces to $\mathrm{Sh} \sim (\mathrm{Sc}\mathrm{Gr}_c)^{1/5}$.

It is easy to see that in order that with $T_w > t_0$ an ascending flow be incident on the lower surface of the disk (or with $T_w < T_0$ a descending flow is incident on its top surface), the inequality

$$\mathrm{Pr}^{-2/3}+\varkappa\,\mathrm{Sc}^{-2/3}>0. \qquad (16)$$

must hold.

When the opposite inequality holds the convective flow realized corresponds to flow of liquid away from the central part of the surface; this is shown in Fig. 1b for the bottom surface of the disk. This situation is best analyzed with the help of the ideas developed in [16-19], according to which the boundary-layer approximation is employed to analyze the converging flow, converging toward the center of the disk. A hydrodynamic boundary layer arises in this case on the outer edge of the disk, and near the center of the disk the boundary layer becomes detached. The fact that the indicated approximation becomes incorrect in the central region is not very significant for evaluating the thermal and diffusion flows, since they become degenerate in this region.

We shall change the sign of the velocity component u, assuming it positive for a flow directed toward the center of the disk, and we introduce the variable $x = R - r$. The equation of continuity can then be represented in the form

$$\frac{\partial}{\partial x}[(R-x)u]+\frac{\partial}{\partial z}[(R-x)v]=0. \qquad (17)$$

The form of the other equations in (1) with $\partial/\partial r$ replaced with $\partial/\partial x$ remains unchanged under the indicated transformations, and the boundary conditions in the region $0 \le x \le R$ are identical to those in (2). We introduce the stream function in this case in accordance with (17):

$$(R-x)\,u = \partial\psi/\partial z,\ (R-x)\,v = -\,\partial\psi/\partial x, \tag{18}$$

and we retain for θ and ϕ the previous definitions in (4) and we perform the substitution of the variables ψ and p using the formulas

$$\begin{aligned}\psi &= R\,[g\,|\beta_T|\,|T_w - T_0|]^{1/5}(\nu x)^{3/5} f(\xi,\ \eta),\\ p &= \rho_0\,[g\,|\beta_T|\,|T_w - T_0|]^{4/5}(\nu x)^{2/5} G(\xi,\ \eta),\end{aligned} \tag{19}$$

and in addition we introduce new independent variables as follows:

$$\xi = x/R,\ \eta = [g\,|\beta_T|\,|T_w - T_0|]^{1/5}(\nu x)^{2/5} z. \tag{20}$$

The problem for the dimensionless stream function, pressure, temperature, and concentration acquires the following form after the transformations:

$$\begin{gathered}(1-\xi)^2 f'''_{\eta^3} + (1-\xi)\left(\frac{3}{5} f f''_{\eta^2} - \frac{1}{5} f'^2_{\eta}\right) - (1-\xi)^3 \times\\ \times\left[\frac{2}{5}(G - \eta G'_{\eta}) - \xi G'_{\xi}\right] + (1-\xi)\,\xi\,(f''_{\eta^2} f'_{\xi} - f'_{\eta} f''_{\xi\eta}) - \xi f'^2_{\eta} = 0,\ G'_{\eta} = \theta + \varkappa\varphi,\\ \xi f'_{\eta}\frac{\partial\theta}{\partial\xi} - \frac{3}{5} f\frac{\partial\theta}{\partial\eta} - \xi f'_{\xi}\frac{\partial\theta}{\partial\eta} = \frac{1-\xi}{\mathrm{Pr}}\,\frac{\partial^2\theta}{\partial\eta^2},\\ \xi f'_{\eta}\frac{\partial\varphi}{\partial\xi} - \frac{3}{5} f\frac{\partial\varphi}{\partial\eta} - \xi f'_{\xi}\frac{\partial\varphi}{\partial\eta} = \frac{1-\xi}{\mathrm{Sc}}\,\frac{\partial^2\varphi}{\partial\eta^2},\\ f(\xi,\ 0) = f'_{\eta}(\xi,\ 0) = f'_{\eta}(\xi,\ \infty) = 0,\ G(\xi,\ \infty) = 0,\\ \theta(\xi,\ 0) = \varphi(\xi,\ 0) = 1,\ \theta(\xi,\ \infty) = \varphi(\xi,\ \infty) = 0.\end{gathered} \tag{21}$$

It is natural to seek the solution of the problem (21) in the form of a series in powers of the variable ξ, and in addition in the region near the edge of the disk, where heat and mass exchange with the liquid predominantly occurs, to a first approximation it is sufficient to calculate the first terms of these series, which corresponds to replacing (21) by a self-similar problem. For the indicated first terms we obtain the problem

$$\begin{gathered}f''' + \frac{3}{5} f f'' - \frac{1}{5} f'^2 = \frac{2}{5}(G - \eta G'),\ G' = \theta + \varkappa\varphi,\\ \theta'' + \frac{3}{5}\mathrm{Pr}\, f\theta' = 0,\ \varphi'' + \frac{3}{5}\mathrm{Sc}\, f\varphi' = 0,\\ f(0) = f'(0) = 0,\ \theta(0) = \varphi(0) = 1,\\ G(\infty) = 0,\ f'(\infty) = 0,\ \theta(\infty) = \varphi(\infty) = 0,\end{gathered} \tag{22}$$

which differs from (5) only by the coefficients. The problem (22) can be analyzed completely analogously to the preceding problem. For θ and φ the formulas in (7) remain valid, but for the dimensionless thicknesses of the thermal and diffusion boundary layers in this case we have

$$\eta_T = 1{,}01\,(10/\alpha\,\mathrm{Pr})^{-1/3},\ \eta_c = 1{,}01\,(10/\alpha\,\mathrm{Sc})^{-1/3}. \tag{23}$$

Using once again the method of joined asymptotic expansions, we obtain the following expression for the coefficient

$$\alpha = 0{,}80\left(\frac{1}{\mathrm{Pr}^{2/3}} + \frac{\varkappa}{\mathrm{Sc}^{2/3}}\right)^{3/5}, \tag{24}$$

replacing (11). Omitting the intermediate calculations we present the final expressions for the Nusselt and Sherwood numbers, replacing (13) and (15):

$$\mathrm{Nu} = 0{,}65\,\mathrm{Pr}^{1/3}\left(\frac{1}{\mathrm{Pr}^{2/3}} + \frac{\varkappa}{\mathrm{Sc}^{2/3}}\right)^{1/5}\mathrm{Gr}_T^{1/5},$$
$$\mathrm{Sh} = 0{,}65\,\mathrm{Sc}^{1/3}\left(\frac{1}{\mathrm{Sc}^{2/3}} + \frac{1}{\varkappa\,\mathrm{Pr}^{2/3}}\right)^{1/5}\mathrm{Gr}_c^{1/5}. \tag{25}$$

The character of the dependence of these numbers on the parameters is identical for both types of flows, but the numerical coefficients are different.

It is not difficult to study using analogous methods the problem of combined thermal-concentration convection at a vertical plate, when the cases of ascending and descending flows are symmetric (see Figs. 1c and d). Omitting the details of the calculations we present the final formulas for the Nusselt and Sherwood numbers:

$$\mathrm{Nu} = 0{,}662\,\mathrm{Pr}^{1/3}\left|\frac{1}{\mathrm{Pr}^{1/3}} + \frac{\varkappa}{\mathrm{Sh}^{1/3}}\right|^{1/4}\mathrm{Gr}_T^{1/4},$$
$$\mathrm{Sh} = 0{,}662\,\mathrm{Sc}^{1/3}\left|\frac{1}{\mathrm{Sc}^{1/3}} + \frac{1}{\varkappa\,\mathrm{Pr}^{1/3}}\right|^{1/4}\mathrm{Gr}_c^{1/4}, \tag{26}$$

where the thermal and concentration Grashof numbers are determined, as before, by the relations (3). In the limit $\kappa \to 0$ or $\kappa \to \infty$ the formulas (26) transform into the well-known formulas $\mathrm{Nu} \sim (\mathrm{PrGr_T})^{1/4}$ and $\mathrm{Sh} \sim (\mathrm{ScGr_C})^{1/4}$; the exact value of the coefficient in (26), obtained with the help of the numerical solution of the problem, is equal to 0.67 [2], which differs insignificantly from the value presented.

Thus the existence of a mass exchange process, accompanying heat exchange between the surfaces and the liquid, can both increase and decrease the effective heat flux, depending on the sign of the ratio κ. An analogous conclusion can also be drawn for the effect of thermal effects on the convective diffusion flow. Moreover, with $\kappa = -(\mathrm{Sc/Pr})^{2/3}$ for horizontal and $\kappa = -(\mathrm{Sc/Pr})^{1/3}$ for vertical surfaces the direction of convective motion itself, generated by the nonuniformity of the density field, can be reversed. The change in the direction of flow corresponds to vanishing of Nu and Sh; this reflects the complete stopping of the convective transport.

We shall evaluate the possibility of a change in the convection regime and the corresponding vanishing of the heat and mass fluxes for the situation when the concentration differential between the surface and the volume of the liquid is assumed to be given, while the temperature differential is regarded as a variable. Thus for the system 1.5N H_2SO_4 + $CuSO_4$ we have the following estimates [20, 21]:

$$\beta_c \approx 1{,}5 - 2{,}0\cdot 10^{-4}\ \mathrm{m^3/mole}\ \beta_T \approx 10^{-3}\ 1/°\mathrm{C}.$$

If it is assumed that c_w corresponds to saturation, then $c_w - c_0 \approx 10 - 10^2$ mole/m³ [21]; in addition [20, 21]:

$$\frac{\mathrm{Sc}}{\mathrm{Pr}} \approx \frac{0.16\cdot 10^{-6}}{0.6\cdot 10^{-9}} = 267.$$

The critical values of $|\kappa|$ for horizontal and vertical surfaces are equal to 41.5 and 6.44, respectively. The values of $|T_w - T_0|$, corresponding to a change in the convection regime, in the cases indicated are equal to 0.1-0.5 and 0.25-3°C, i.e., they are very small. It follows from here that the most insignificant thermal effect from heterogeneous reactions can already radically change the conditions for the flow of these reactions; this is undoubtedly important both for interpreting experimental data and for organizing these reactions under technological conditions.

In conclusion we note that the condition $\mathrm{Pr} \gg 1$ for the analysis presented to be approximately valid turns out to be very restrictive. However the procedure employed to solve the problems studied can in principle be extended, which makes it possible to calculate without any special difficulties the terms in the expansions of Nu and Sh in negative fractional powers of the parameters Pr and Sc.

NOTATION

a, thermal diffusivity; c, concentration; d, diffusion coefficient; f and G, dimensionless stream function and pressure; g, acceleration of gravity; j, local flux; m and n, coefficients in (10); p, pressure; R, radius of the disk; r, radial coordinate; T, temperature;

u and v, velocity components; $x = R - r$; z, vertical coordinate; β_T and β_C, inverse coefficients of thermal and concentration expansion; α, coefficient in (9); η and ξ dimensionless variables defined in (4) and (20); φ, parameter defined in (6); ν, kinematic viscosity; θ and φ, dimensionless temperature and concentration; ρ, density; ψ, stream function; Nu, Sh, Gr, Pr, and Sc, Nusselt, Sherwood, Grashof, Prandtl, and Schmidt numbers. The indices 0 and w refer to states in the volume of the liquid and at the surface, respectively.

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